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**Question**

Cthulhu Publishing, a well-recognized distributor of video games, uses several distribution centers to gather its game inventory and then supplies them to the retail stores spread all over the country. Both efficient transportation and cost management are key factors in keeping competitive pricing and ensuring that the newest games are delivered on time to enthusiastic recipients.

Polypus Publishing has five main distribution centers located in different cities, each storing a variety of video games:

* Distribution Center D1 (Isengard): Supply of 220 video game units
* Distribution Center D2 (Tatooine): Supply of 170 video game units
* Distribution Center D3 (Whiterun): Supply of 195 video game units
* Distribution Center D3 (Night City): Supply of 160 video game units
* Distribution Center D3 (Limgrave): Supply of 185 video game units

Cthulhu Publishing supplies its video games to five major retail stores, each requiring a steady supply to meet customer demand:

* Retail Store R1 (Mordor): 205 video game units required
* Retail Store R1 (Alderaan): 210 video game units required
* Retail Store R1 (Morrowind): 180 video game units required
* Retail Store R4 (New Vegas): Requires 165 units of video games.
* Retail Store R4 (Leyndell): Requires 170 units of video games.

The cost of transporting one unit of video games from each distribution center to each retail store varies based on distance and logistical factors. The transportation cost matrix is below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 | 5 | 8 | 6 | 9 | 220 |
| **D2** | 6 | 3 | 7 | 4 | 5 | 170 |
| **D3** | 5 | 6 | 7 | 2 | 8 | 195 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 205 | 210 | 180 | 165 | 170 |  |

Cthulhu Publishing needs to determine the optimal number of units of video games to transport from each distribution center to each retail store to minimize the total transportation cost while meeting the supply and demand constraints.

Let xij represent the number of units transported from distribution center Di to retail store Rj.

Objective Function**: minZ = 2 \* x11 ​+ 5 \* x12 ​+ 8 \* x13​ + 6 \* x14​ + 9 \* x15​ + 6 \* x21 ​+ 3 \* x22​+ 7 \* x23​ + 4 \* x24​ + 5 \* x25 + 5 \* x31 ​+ 6 \* x32 ​+ 2 \* x33 ​+ 7 \* x34 ​+ 8 \* x35 ​+ 9 \* x41 ​+ 4 \* x42 ​+ 6 \* x43 ​+ 3 \* x44 ​+ 7 \* x45 ​+ 7 \* x51 ​+ 8 \* x52 ​+ 4 \* x53 ​+ 5 \* x54 ​+ 6 \* x55​**

**Constraints**

* From Distribution Center D1 (Isengard): x11 ​+ x12 ​+ x13​ ​+ x14 + x15 ≤ 220
* From Distribution Center D2 (Tatooine): x21​ + x22 ​+ x23 ​​+ x24 + x25 ≤ 170
* From Distribution Center D3 (Whiterun): x31​ + x32 ​+ x33 ​+ x34 + x35 ​≤ 195
* From Distribution Center D3 (Night City): x41​ + x42 ​+ x43 ​+ x44 + x45 ​≤ 160
* From Distribution Center D3 (Limgrave): x51​ + x52 ​+ x53 ​+ x54 + x55 ​≤ 185
* To Retail Store R1 (Mordor): x11​ + x21​ + x31​ + X41 + x51 ≥ 205
* To Retail Store R2 (Alderaan): x12 ​+ x22 ​+ x32​ + X42 + x52 ≥ 210
* To Retail Store R3 (Morrowind): x13 ​+ x23​ + x33 + X43 + x53 ​≥ 180
* To Retail Store R3 (New Vegas): x14 ​+ x24​ + x34 ​+ X44 + x54 ≥ 165
* To Retail Store R3 (Leyndell): x15 ​+ x25 + x35 + X45 + x55 ​≥ 170
* xij​ ≥ 0 for all i,j

Determine the values of xij that minimize the total transportation cost while satisfying the supply limits of the distribution centers and the demand requirements of the retail stores.

**Solution**

North-West Corner Method is used.

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| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 | 8 | 6 | 9 | 15 |
| **D2** | 6 | 3 | 7 | 4 | 5 | 170 |
| **D3** | 5 | 6 | 7 | 2 | 8 | 195 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 210 | 180 | 165 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 | 7 | 4 | 5 | 170 |
| **D3** | 5 | 6 | 7 | 2 | 8 | 195 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 210 | 180 | 165 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 | 7 | 2 | 8 | 195 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 25 | 180 | 165 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 | 2 | 8 | 170 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 0 | 180 | 165 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 0 |
| **D4** | 9 | 4 | 6 | 3 | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 0 | 10 | 165 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 0 |
| **D4** | 9 | 4 | 6 (10) | 3 | 7 | 150 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 0 | 0 | 165 | 170 |  |

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| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 0 |
| **D4** | 9 | 4 | 6 (10) | 3 (150) | 7 | 0 |
| **D5** | 7 | 8 | 4 | 5 | 6 | 185 |
| **Demand** | 0 | 0 | 0 | 15 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 0 |
| **D4** | 9 | 4 | 6 (10) | 3 (150) | 7 | 0 |
| **D5** | 7 | 8 | 4 | 5 (15) | 6 | 170 |
| **Demand** | 0 | 0 | 0 | 0 | 170 |  |

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| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 0 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 0 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 0 |
| **D4** | 9 | 4 | 6 (10) | 3 (150) | 7 | 0 |
| **D5** | 7 | 8 | 4 | 5 (15) | 6 (170) | 0 |
| **Demand** | 0 | 0 | 0 | 0 | 0 |  |

**Feasible Solution:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **D/R** | **R1** | **R2** | **R3** | **R4** | **R5** | **Supply** |
| **D1** | 2 (205) | 5 (15) | 8 | 6 | 9 | 220 |
| **D2** | 6 | 3 (170) | 7 | 4 | 5 | 170 |
| **D3** | 5 | 6 (25) | 7 (170) | 2 | 8 | 195 |
| **D4** | 9 | 4 | 6 (1) | 3 (150) | 7 | 160 |
| **D5** | 7 | 8 | 4 | 5 (15) | 6 (170) | 185 |
| **Demand** | 205 | 210 | 180 | 165 | 170 |  |

**Min total cost =** (2 × 205) + (5 × 15) + (3 × 170) + (6 × 25) + (7 × 170) + (6 × 10) + (3 × 150) + (5 × 15) + (6 × 170) = 3940